Note that, if the Hugoniot stress-strain relation is a straight line, $-d\sigma/dV = \sigma/(V_0 - V)$, then errors in particle velocity do not produce errors in the stress-strain curve.

In the present experiments the curves are nearly straight and, approximately,

$$\Delta\sigma/\sigma = 2\ \delta U/U$$

The error in stress at a given volume is thus twice the error in shock velocity at a given particle velocity. The corresponding error in the fourth-order elastic coefficient is

$$\frac{dc_{1111}}{c_{1111}} = \left[\frac{6c_{11}}{c_{1111}N_1^{\,2}} + \frac{3c_{111}}{c_{1111}N_1} + 1\right]\frac{d\sigma}{\sigma}$$

Taking the error in shock velocity to be $\pm 1\%$, we find the corresponding fractional change in $c_{1111}$ to be

$$dc_{1111}/c_{1111} = 9\ d\sigma/\sigma = \pm 18\%$$

For Z-cut crystals the result is

$$dc_{3333}/c_{3333} = \pm 10\%$$

The total precision of the fourth-order constants from all sources is, therefore, estimated to be

$$dc_{1111}/c_{1111} = \pm 20\%$$

and

$$dc_{3333}/c_{3333} = \pm 20\%$$

## Conclusions

The behavior of quartz under shock loading conditions is very much different from that of metals, as was pointed out by Wackerle. The elastic precursor waves are an order of magnitude higher and, correspondingly, so are the shear stresses. The curve labelled X in Figure 9 is the normal stress component (based on constants to third-order) across a plane perpendicular to the shock front when the shock propagates in the Z direction. The maximum stress difference is seen to exceed 100 kb. This is of the same order of magnitude as the effective shear modulus; consequently, it appears that quartz momentarily exhibits theoretical yield strength under dynamic conditions.

That cohesion of the material is destroyed upon yielding is indicated by the close agreement of the second shocked states with Bridgman's hydrostatic data. There is no indication of a residual shear stress, in contrast to the case for metals [*Fowles*, 1961*a*].

The pronounced stress relaxation shown by the observed variation in amplitude of the elastic waves and the apparent dependence on the final pressure is quantitatively larger than for metals, although it is similar qualitatively.

Evidently, shock wave methods provide a valuable supplement to low-pressure acoustic measurements in determining higher-order elastic constants, as least for ceramic type materials that sustain large-amplitude elastic waves. Shock waves are inherently more suitable for higher-pressure measurements than acoustic methods are, but they are less suitable for the high-precision, low-pressure measurements required to evaluate second-order constants. To what extent shock wave techniques are capable of measuring coefficients other than the principal coefficients, i.e., the directions for which the elastic wave is purely longitudinal, requires additional study.

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